

Introduction to Mathematics for Software Engineering

SET07106 Mathematics for Software Engineering

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Outline

CAS Software

Tasks

Prime numbers

Infinity

Using a CAS

What kind of maths skills
are needed by software engineers?

Computer Algebra System (CAS) software

- ▶ A software program that facilitates symbolic mathematics.
- ▶ Manipulation of mathematical expressions in symbolic form.

Examples of CAS software

- ▶ Commercial:
 - ▶ MATLAB (developed by The MathWorks)
 - ▶ Mathematica (Stephen Wolfram)
 - ▶ Maple
- ▶ Free: Sage (a front-end to several free CAS)

Tasks for CAS software

- ▶ simplification, modification of and substitution in expressions
- ▶ solution of equations, differentiation, integration
- ▶ series operations and limits
- ▶ matrix operations
- ▶ statistical and numerical operations
- ▶ logical operations, theorem proving
- ▶ plotting of graphs

Mathematical tasks for software engineers

Since CAS exists, what are the tasks that are left for humans to do?

Problem solving example

How high above ground are we in this lecture room on the H floor?

Can you estimate?

If you were given half an hour time and access to some tools, how would you determine the height?

Mathematical tasks for software engineers

- ▶ Modelling of problems and solutions
- ▶ Identifying the type of a problem in order to find the type of the solution
- ▶ Estimation of the results in order to verify that CAS result is correct
- ▶ Testing of results
- ▶ Mathematical thinking

Matching types of problems with types of solutions

type of problem	type of solution
?	differentiation
?	integration
?	functions
?	equations
sudoku	?
encryption	?
expert systems, AI	?
risk, life insurance	?

Estimation

- ▶ $\frac{1}{x}, x \rightarrow \infty$
- ▶ $X^2, x \rightarrow \infty$
- ▶ What is larger 2^{10} or 10^2 ?
- ▶ How much is 15% of 50?
- ▶ What is more probable: to win (a big prize) in the lottery or to have a car accident?
- ▶ What is the circumference of the Earth?
- ▶ How many times can you fold an A4 sheet of paper in half?
If the paper is 0.1 mm thick and you were able to fold it 40 times, how thick would it be?

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Example 1: prime numbers

Numbers that are only divisible by 1 and by themselves.

2, 3, 5, 7, 11, 13, 17, ...

An algorithm for finding prime numbers

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

An algorithm for finding prime numbers (2)

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

An algorithm for finding prime numbers (3)

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

An algorithm for finding prime numbers (5)

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

An algorithm for finding prime numbers (7)

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

This algorithm is called: the *Sieve of Eratosthenes*.

How many prime numbers are there?

How many prime numbers are there?

$$(1 * 2 * 3) + 1 = 7$$

$$(1 * 2 * 3 * 5) + 1 = 31$$

$$(1 * 2 * 3 * 5 * 7) + 1 = 211$$

$$(1 * 2 * 3 * 5 * 7 * 11) + 1 = 2311$$

Not all numbers in this list are prime numbers:

$$(1 * 2 * 3 * 5 * 7 * 11 * 13) + 1 = 30031 = 59 * 509$$

But there cannot be a largest prime.

Example 2: infinity

How many ...

- ▶ integers
- ▶ even numbers
- ▶ rational numbers
- ▶ real numbers

How many rational numbers?

$1/1$	$1/2$	$1/3$	$1/4$	$1/5$	$1/6$	$1/7$...
$2/1$	$2/2$	$2/3$	$2/4$	$2/5$	$2/6$	$2/7$...
$3/1$	$3/2$	$3/3$	$3/4$	$3/5$	$3/6$	$3/7$...
$4/1$	$4/2$	$4/3$	$4/4$	$4/5$	$4/6$	$4/7$...
$5/1$	$5/2$	$5/3$	$5/4$	$5/5$	$5/6$	$5/7$...
$6/1$	$6/2$	$6/3$	$6/4$	$6/5$	$6/6$	$6/7$...
$7/1$	$7/2$	$7/3$	$7/4$	$7/5$	$7/6$	$7/7$...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	

How many rational numbers?

$1/1$	$1/2$	$1/3$	$1/4$	$1/5$	$1/6$	$1/7$...
$2/1$	$2/2$	$2/3$	$2/4$	$2/5$	$2/6$	$2/7$...
$3/1$	$3/2$	$3/3$	$3/4$	$3/5$	$3/6$	$3/7$...
$4/1$	$4/2$	$4/3$	$4/4$	$4/5$	$4/6$	$4/7$...
$5/1$	$5/2$	$5/3$	$5/4$	$5/5$	$5/6$	$5/7$...
$6/1$	$6/2$	$6/3$	$6/4$	$6/5$	$6/6$	$6/7$...
$7/1$	$7/2$	$7/3$	$7/4$	$7/5$	$7/6$	$7/7$...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	

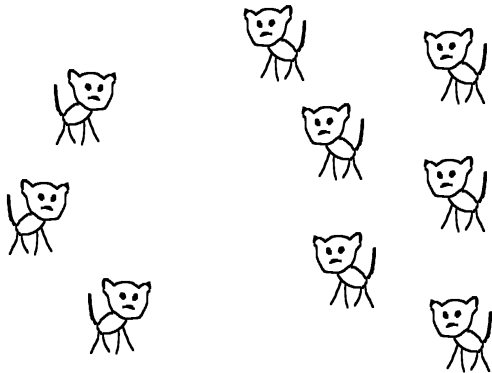
How many integers, even numbers, rational numbers?

1 2 3 4 5 6 7 8 ...

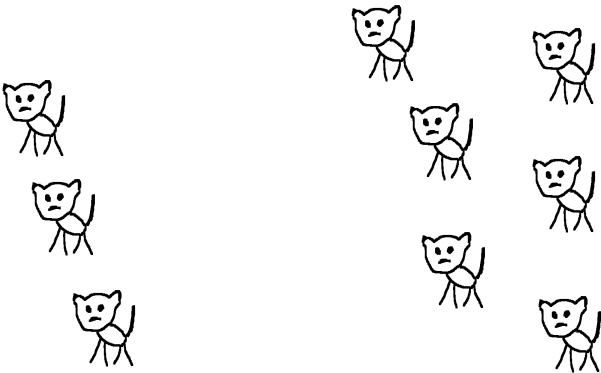
2 4 6 8 10 12 14 16 ...

1/1 1/2 2/1 3/1 2/2 1/3 1/4 2/3 ...

Intuition for numbers



Intuition for numbers



Intuition for numbers

Intuitively: there should be twice as many integers than there are even numbers.

But there are not!

Integers, even numbers, odd numbers, rational numbers are all *countably infinite*.

Real numbers are not countable. There are more real numbers than integers.

Numbers in programming languages

How large are integers and real numbers in a computer?

How much can be computed?

The SAGE open-source CAS

A collection of tools all accessible through a Python interface.

- ▶ SymPy and Maxima for calculus
- ▶ SciPy and NumPy for optimisation, linear algebra, integration, matrices
- ▶ FLINT, NTL and Pari for Number Theory
- ▶ PyCrypto and OpenCDK for Cryptography
- ▶ GD - Dynamic graphics generation tool
- ▶ IML Integer Matrix Library
- ▶ NetworkX Graph theory
- ▶ SQLAlchemy and Sqlite for relational database and algebra
- ▶ ... and more!

Why Python?

- ▶ Python is a multi-purpose language which has many libraries for mathematics, language, etc, but can also do web pages, AI, Unix scripting and much more.
- ▶ Python is pre-installed on Linux and Mac OS.
- ▶ Scripting languages are easy to learn and use.
- ▶ Only a very small subset of Python is needed for this module.
- ▶ Python has a datatype for Sets.
- ▶ Python supports some forms of functional programming

One notable difference between Python and other languages: Python uses code indentation instead of curly brackets `{}`.

Code indentation

```
for ... :  
    while ... :  
        if ... :  
            do something  
        if ... :  
            do something
```

Elements and data types

Data types in programming languages: integer, float, string ...
The data type determines the operations that are available.

In mathematics: elements are usually part of a mathematical structure. This is usually a class with some operations:

- ▶ integers with $+$, $-$, $*$
- ▶ rational numbers with $+$, $-$, $*$, $/$
- ▶ real numbers with $+$, $-$, $*$, $/$
- ▶ sets with set operations
- ▶ graphs with graph operations

Division in programming languages

division	$21 / 10 == 2.1$
integer division	$21 // 10 == 2$
remainder	$21 \% 10 == 1$

Arithmetical operator precedence

1. exponentiation	**
2. multiplication, division, remainder	*, /, //, %
3. addition, subtraction	+, -
4. comparison	==, <, <=, >, >=, !=

What is

$$5 - 2^3 * 3 + \frac{6}{2}$$

$$(5 - 2**3 * 3 + 6/2)$$

Variables

Programming language variables:

```
n1 = 4
```

```
n2 = 5
```

```
result = n1 * n2
```

Mathematical variables:

```
x = Symbol('x')
```

```
y = Symbol('y')
```

```
x**2 + 5 * y - 2
```

$$x^2 + 5y - 2$$