

Graphs

SET07106 Mathematics for Software Engineering

School of Computing
Edinburgh Napier University
Module Leader: Uta Priss

2010

Outline

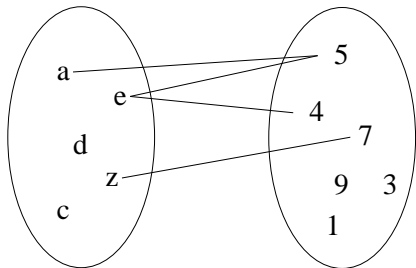
Graphs

Eulerian and Hamiltonian

Applications

Graph layout software

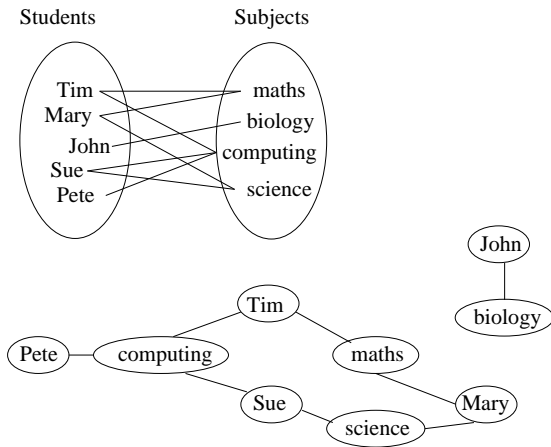
Binary relations



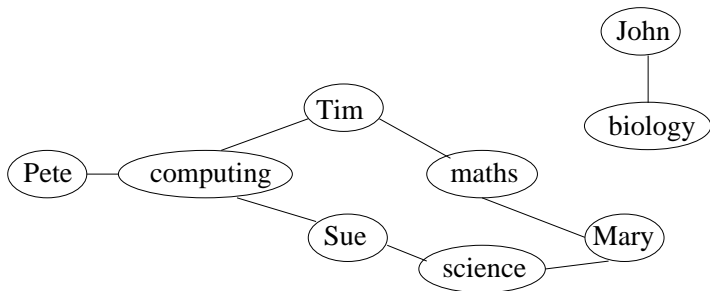
	1	3	4	5	7	9
a				X		
c						
d						
e			X	X		
z					X	

$\{ (a,5), (e,5), (e,4), (z,7) \}$

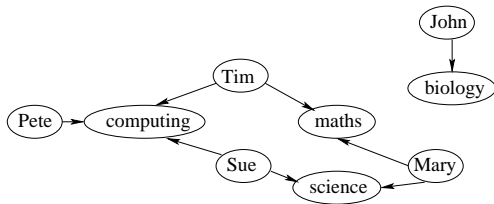
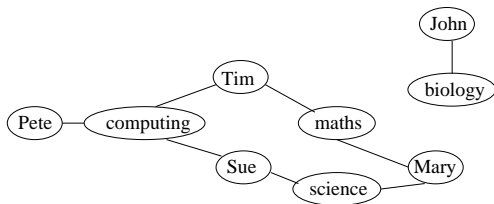
A binary relation can be represented as a graph



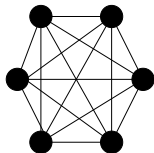
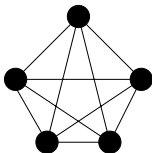
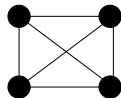
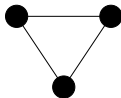
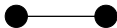
A graph consists of nodes (vertices) and edges



Undirected and directed graphs



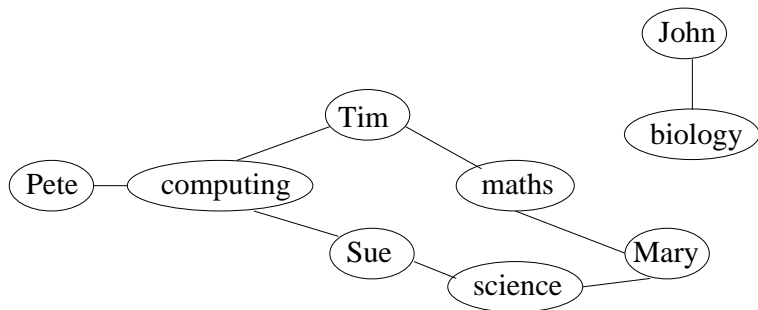
The complete graphs for $n \leq 6$



Exercises

- ▶ Draw the complete graph with 7 nodes.
- ▶ How many edges does every node have in a complete graph?
- ▶ How many edges does a complete graph have?

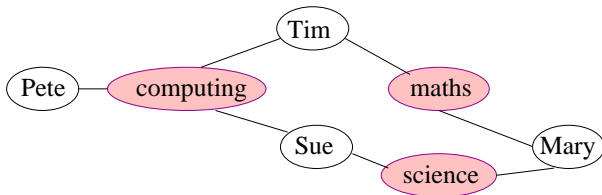
A graph can be connected or disconnected



A bipartite graph has two sets of nodes

Edges are from one set of nodes to the other.

There are no edges within the same set of nodes.

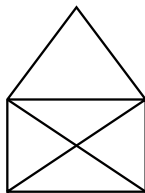


Exercise

Are there any complete graphs which are bipartite?

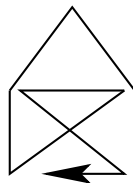
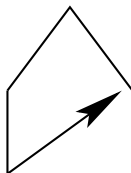
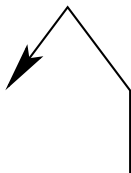
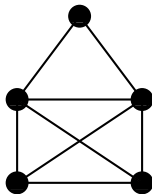
Can you draw this figure ...

in one go without starting and stopping in between?



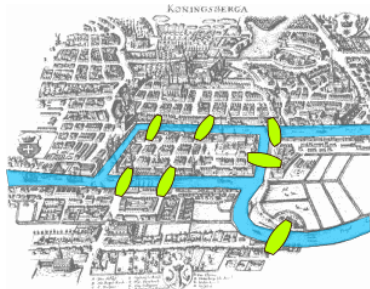
Eulerian path

Each edge is visited once.



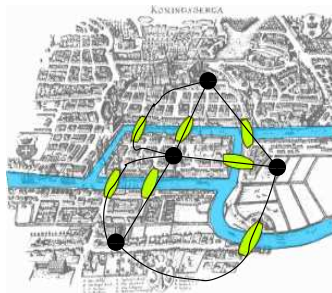
Seven bridges of Königsberg (1735)

Walk through the city and cross each bridge exactly once.



Seven bridges of Königsberg

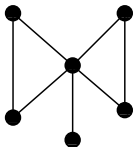
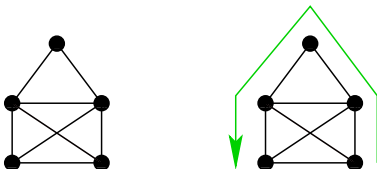
This is graph problem.



Euler asserted that a graph has a Eulerian path if the graph is connected and has either no or two nodes with an odd number of edges.

Hamiltonian path

Each node is visited once.



Travelling salesman problem

What is the shortest path for a salesman to visit a given set of cities?

A problem of

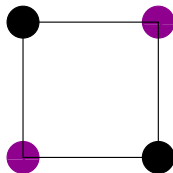
- ▶ optimisation
- ▶ planning

A graph where the edges are labelled with the distances between the cities.

Among all the Hamiltonian paths, find the one which minimises distances.

Sudoku

1	2
2	1

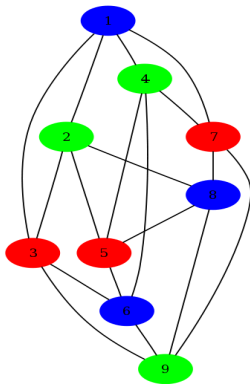


A bipartite graph.

Sudoku

1	2	3
2	3	1
3	1	2

1	2	3
4	5	6
7	8	9



A tripartite graph.

6 degrees of separation

Do you know the Prime Minister?

Do you know someone who knows the Prime Minister?

Do you know someone who knows someone who knows the PM?

...

The claim: everybody is connected to everybody else by at most 6 degrees of separation.

⇒ It is a **small world**.

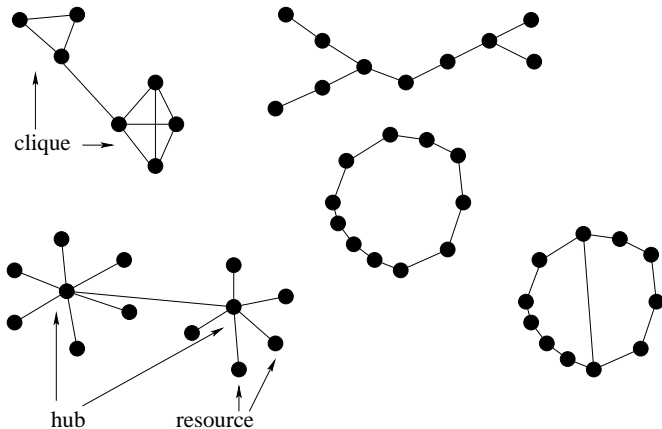
Small-world effect

Every node can be reached by every other node by a short path.

Examples:

- ▶ Social networks
- ▶ Internet
- ▶ Road maps
- ▶ Electric power grids

Which of these have a small-world effect?



Small-world networks

- ▶ Small world effect: small average node-to-node distance (shortest path length)
- ▶ Clustering coefficient that is larger than the clustering coefficient of a random graph with the same number of nodes and edges.

Having a large clustering coefficient means that “the people you know also know each other”.

Other graph applications

- ▶ Links between webpages
- ▶ Sitemaps
- ▶ Flow charts, UML diagrams
- ▶ Database schemata, ER diagrams
- ▶ Class hierarchies
- ▶ Web site paths traversed by users
- ▶ XML tree structures and DTDs

Picture of a Null Graph:

Harary and Read (1973): Is the Null Graph a Pointless Concept?

“The graph with no points and no lines is discussed critically. ...
No conclusion is reached.”

The question is not whether it exists, but whether there is a point
in it.

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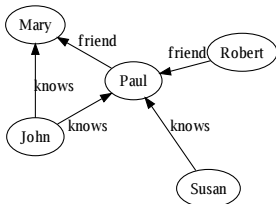
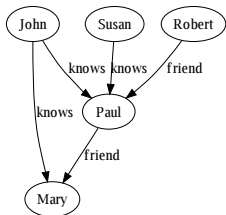
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From Wolfram MathWorld:

“the only good null graph is a dead null graph”

Graphs are special kinds of vector graphics

- ▶ Moving or removing a node affects its edges.
- ▶ Graph editors provide graph layout algorithms.



Graph layout software/editors

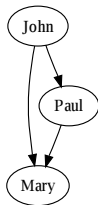
- ▶ TouchGraph, spring embedder algorithms
- ▶ Java toolkits: Prefuse, ...
- ▶ Graphviz: open source graph visualisation software

Graphviz

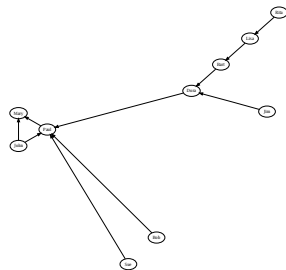
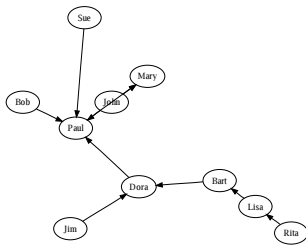
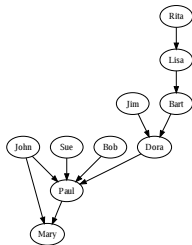
- ▶ www.graphviz.org
- ▶ Directed and undirected graphs.
- ▶ Graph layouts: hierarchies, spring, radial, circular.
- ▶ Simple text-based format (called “dot format”).
- ▶ APIs for different programming languages exist.
- ▶ Many output formats: gif, jpg, svg, pdf, ...

The “dot format”

```
digraph names {  
  node0 [label=' 'John' ']  
  node1 [label=' 'Mary' ']  
  node2 [label=' 'Paul' ']  
  node0 -> node1  
  node0 -> node2  
  node2 -> node1  
}
```



Hierarchical, radial, circular layouts:



Spring layouts:

