Conceptual Schemata as a Means for Structuring Teaching Materials

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Effective teaching requires study materials to be well-structured, provide adequate detail, support multiple modes of representation and to progress from prerequisite to more advanced concepts. Methods such as "Decoding the Disciplines" [4] and APOS-Theory [1] present strategies for conceptual development of teaching materials, but without determining a formal method. The approach suggested in this abstract builds on APOS-Theory, Formal Concept Analysis (FCA) [2], Conceptual Graphs (CG) [7] and Semiotic-Conceptual Analysis (SCA) [5] and is aimed at developing a practical, but formal method. The core notion of *conceptual schema* is influenced both by APOS-Theory as well as Lakoff's [3] image schemata.

In FCA, a *formal concept* is defined by an extension (e.g., set of exemplars) and an intension (e.g., a conjunction of predicates) described with logical precision within a fixed formal context. For example, the mathematical concept of "function" can be intensionally defined as "relation(x) $\wedge$ left_total(x) $\wedge$ right_unique(x)" or by any logically equivalent statement independently of the exact vocabulary that is used. Concepts relating to natural language words may not have precise, logical, vocabulary-independent definitions and may be better characterised using a *conceptual schema* (also referred to just as *schema*) which is less formal and closer to a natural language description. Some of the words or phrases used in a schema are considered *head representamens* in this abstract. Head representamens denote the concepts that are to be described by a set of schemata. A schema consists of a set of head representamens and relations amongst them such as part-whole, exemplar-attribute, subconcept-superconcept and other relations. Relations are indicated by function words, syntactic features or formally represented as CGs, FCA lattices or similar. Head representamens from one schema can also point to head representamens from other schemata. The semantics of the schemata and their relations can be provided by mappings (or interpretations) into a more formal structure such as FCA [6].

While conceptual schemata provide a means for structuring teaching materials, it is also important to consider how materials are represented. In domains such as mathematics, different *representamen types* [5] may be available, such as graphs, diagrams or formulas instead of just words. Representamen types can be chosen to optimise representations: being efficient and comprehensive but also allowing to highlight different modes and points of view.

The remainder of this abstract discusses the use of head representamens, schemata and representamen types for teaching materials relating to the concept of a mathematical function. Fig. 1 displays a schema for a head representamen "(mathematical) function". The left side depicts a part-whole relation that is graphically represented as a

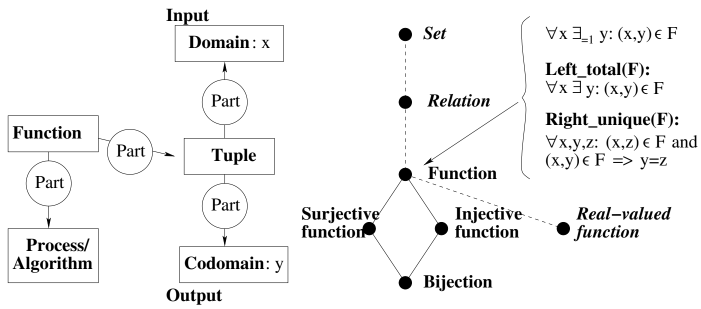
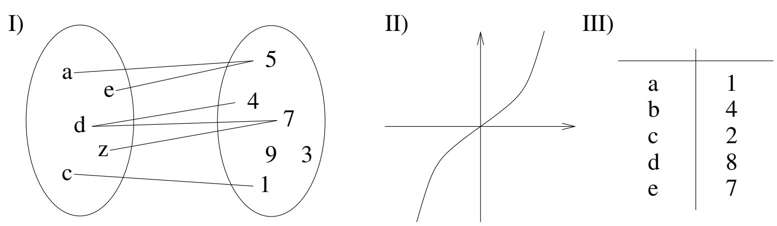


Fig. 1. A conceptual schema for the mathematical head representamen "function"

CG. The middle of Fig. 1 shows a subconcept-superconcept relation as a part of a FCA concept lattice. Edges that are dashed instead of solid indicate pointers to head representamens of other schemata (represented by italic font). The predicates listed on the right belong to the intensions of formal concepts. The words and phrases in bold font are head representamens that are usually contained in the context of functions. All other words and characters are not considered head representamens.



IV) $f : D \to M$
$x \mapsto x^2$ V) $\{(a, 1), (c, 2), (b, 1)\}$ VI) a:=proc(A){return A * A};

Fig. 2. Six different representamen types of "function"

For teaching purposes it is important to identify which head representamens belong to which conceptual schema and which are prerequisite and should be taught first. Prerequisite head representamens tend to be higher up in a concept lattice of a conceptual schema. For example, set and relation are prerequisite for function in Fig. 1. A concept such as "function" tends to be viewable from different angles, for different purposes and in different contexts. In order to fully understand a concept, students need to shift between a variety of representations of it supplied by different representamen types covering different head representamens of the schema.

Fig. 2 displays six representamen types of "function": diagram, graph, table, declaration, set of pairs and code of a programming language. Concept lattices as in Fig. 3

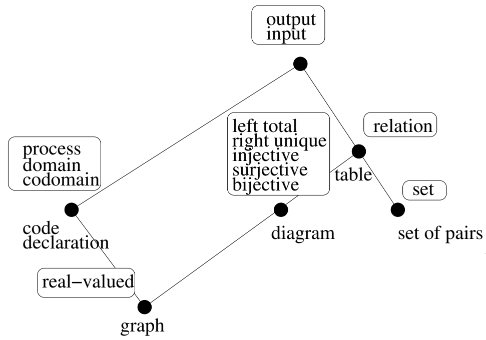


Fig. 3. A part of a concept lattice of representamen types and head representamens

can be generated using FCA software to investigate which concepts (represented by their head representamens) are observable in which representamen types. The representamen type "graph" covers all head representamens except "set" because the fact that functions are sets is not observable from a graph. But graphs are only applicable to functions that are real-valued. Therefore students might conflate the schemata of "function" and "real-valued function". Furthermore, representamen types may invoke misleading schemata based on their 2-dimensional representation. For example, the size of the ellipses in Fig. 2 I) is irrelevant. Thus, teaching with more than one representamen type is essential.

In summary, the method proposed in this abstract is to 1) structure a domain for teaching purposes by identifying head representamens and their conceptual schemata; 2) determine prerequisite relationships amongst schemata; 3) select a set of representamen types that covers all head representamens of all schemata while ensuring that schemata and head representamens are sufficiently discriminated. The steps of this method are supported by FCA software.

References

1. Arnon, I., Cottrill, J., Dubinsky, E., Oktac, A., Roa Fuentes, S., Trigueros, M., Weller, K.: APOS Theory – A framework for research and curriculum development in mathematics education. Springer (2013).
2. Ganter, B.; Wille, R.: Formal Concept Analysis. Mathematical Foundations. Springer (1999).
3. Lakoff, G.: Women, Fire, and Dangerous Things. What Categories Reveal about the Mind. The University of Chicago Press (1987).
4. Middendorf, J.; Pace, D.: Decoding the Disciplines: A Model for Helping Students Learn Disciplinary Ways of Thinking. In: New directions for teaching and learning, 98, p. 1-12 (2004).
5. Priss, U.: Semiotic-Conceptual Analysis: A Proposal. International Journal of General Systems, 46, 5, p. 569-585 (2017).
6. Priss, U.: Modelling Conceptual Schemata with Formal Concept Analysis. In: FCA4AI Workshop, CEUR, 2021. (to appear)
7. Sowa, J.: Conceptual Graphs. In: Foundations of Artificial Intelligence, p. 213-237 (2008).