Using FCA for Modelling Conceptual Difficulties in Learning Processes*

Uta Priss, Peter Riegler, Nils Jensen

Zentrum für erfolgreiches Lehren und Lernen
Ostfalia University of Applied Sciences
Wolfenbüttel, Germany
www.upriss.org.uk, {p.riegler,n.jensen}@ostfalia.de

Abstract. In the natural sciences, mathematics and technical subjects, universities often observe generally low pass rates and high drop out rates in the first years. Many students seem to have conceptual difficulties with technical and mathematical materials. Furthermore, physics education research appears to indicate that even students who are able to pass exams may still not have a good understanding of basic physics concepts. Some researchers use the notion of "misconception" to describe conceptual differences between intuitive notions and accepted scientific notions. A significant body of educational research exists dedicated to overcoming such didactic challenges, but so far not much Formal Concept Analysis (FCA) research has been dedicated to these topics. The aim of this paper is to develop a better understanding of the structure of conceptual difficulties in learning processes using FCA. It is not intended in this paper to develop new educational methods or to collect new data, but instead to analyse existing data and models from an FCA viewpoint.

1 Introduction

Education is an interesting application area for Formal Concept Analysis[1] (FCA) because the analysis, representation and development of conceptual structures - in the mind of the learner, and maybe also of the teacher - is an inherent feature of learning and teaching. Because modern e-learning materials and environments tend to accumulate and provide large amounts of data, any technology, such as FCA, developed for structuring and retrieval of information or semantic, conceptual and ontological analysis is implicitly applicable to learning materials as well.

Rudolf Wille the founder of FCA also pioneered the use of FCA for teaching mathematics (Wille, 1995). He published a number of subsequent papers on mathematics restructuring and education - most of them are more general, of philosophical nature and not specifically about FCA. Otherwise, there do not appear to be significant numbers of FCA publications in the educational domain. Examples of FCA applications in

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[1] Because this conference is dedicated to FCA, this paper does not provide an introduction to FCA. Information about FCA can be found, for example, on-line (http://www.upriss.org.uk/fca/) and in the main FCA textbook by Ganter & Wille (1999).

this area focus on ontological representations, such as the structuring, retrieval and visualisation of learning materials (Lee, 2005) or the development of an ontology-based courseware management system (Tane et al., 2004) which facilitates browsing, querying and clustering of materials and ontology evolution. Other FCA applications relate to the use of FCA with computer algebra systems (Priss, 2010) and to the meta-analysis of learning materials. For example Pecheanu et al. (2011) use FCA to evaluate and compare open source learning platforms.

Apart from the general data analysis and knowledge representation applications, it is of interest to use FCA to directly analyse the cognitive structures involved in learning processes because presumably learning consists of concept formation, ordering and structuring. Applying FCA in this area is not fundamentally different from other applications where different concept lattices might represent the views of different experts except that in teaching there is an expectation that some conceptual structures are correct and some are not and that the conceptual structures of the students are intended to change.

One obvious difficulty is that it is not easy to obtain representations of such cognitive structures. Psychologists have developed methods for eliciting and externally representing mental models. A number of applications of FCA in the psychological domain have been described, for example, by Spangenberg and Wolff (1991). Al-Diban and Ifenthaler (2011) discuss the comparison of two methods for eliciting and analysing mental models of students, one of which uses FCA. These methods build on a qualitative analysis of data, including transcribed and coded textual protocols, and data collected from specific tests where subjects order concepts in if-then relations. A disadvantage of these methods is that it is not clear whether they could be applied to data observed in real teaching situations (instead of collected from tests) because real data might not have sufficient structure and detail. Furthermore at least in the Al-Diban and Ifenthaler study, the focus was on declarative knowledge, that is whether students know certain facts, not so much on degrees of understanding. An advantage of these methods is that, for example, conceptual gaps and differences among different students and between students and teachers can be detected and analysed.

In addition to the analysis of conceptual structures in learning processes, one might also want to model the conceptual space of a domain for teaching purposes. This has been achieved by Falmagne et al. (2006) who describe a "knowledge state" as the set of particular problems a student can answer in a mathematical topic area. Feasible knowledge states are represented with respect to a precedence graph. This graph is a partially ordered set and not a lattice, but it could be embedded into a lattice and thus modelled with FCA. The idea is that knowledge is ordered: if someone masters a certain mathematical problem then that person can also solve problems that are simpler but may still have to learn to solve problems that are more difficult. Because the precedence graph is not a linear order, different students can take different learning paths. The position of the knowledge state of a student in the graph shows exactly which problems the student can attempt to learn to solve next. Furthermore the student's progress can be exactly measured. Establishing a precedence graph for a mathematical topic area which might consist of hundreds of states is labour-intensive but feasible in a commercial environment such as Falmagne et al.'s ALEKS software tool. There are different means for building such a precedence graph: by questioning experts about the difficulties and prerequisites of problems, by analysing student data collected from an e-learning tool or by analysing problem solving processes in the domain.

Currently, ALEKS focuses on mathematics and science topics. It is not clear how far such approaches would be suitable for other non-science domains where it would be difficult to establish a precise ordering of problems. Furthermore, it it may be difficult to evaluate how accurate and useful a particular precedence graph is because user testing of complex e-learning tools is notoriously difficult. If student learning improves while they are using ALEKS, it would be difficult to determine whether that is because of the precedence graph or because of any other of ALEKS's many features.

In summary, general knowledge representation and retrieval aspects of e-learning tools are not any different to such aspects of other textual databases and are covered sufficiently in other domains than educational research. But the analysis of conceptual structures involved in learning processes and the conceptual structuring of domain knowledge for learning purposes is specific to educational research. Currently, FCA appears to be underrepresented in these tasks but it should be very applicable to both of these tasks. It is of particular interest to study differences between the conceptual structures of a learner and of an expert, such as knowledge gaps, discrepancies and misconceptions. A goal for this paper is to involve FCA in the analysis, description and detection of conceptual difficulties, including misconceptions. Section 2 of this paper provides an overview of challenges encountered in teaching conceptually difficult topics. The following three sections show examples of conceptual difficulties for selected mathematical topics: equality in Section 3, translating text into algebraic expressions in Section 4 and the notion of "function" in Section 5. The lattices in these examples are developed from the viewpoint of a teacher who is exploring the difficulties in these areas by modelling formal contexts based on the description of misconceptions in the literature and based on student data.

2 Successful and unsuccessful teaching

Physics Education Research studies the problems students have in acquiring physics concepts. Hake (1998) explains that students have initial common-sense beliefs about 'motion' which are in contradiction to current physics theory and which are not improved on by traditional educational methods. Hestenes et al. (1992) published a Force Concept Inventory (FCI) designed to test the students' conceptual understanding of Newtonian mechanics which can be used before and after a physics course to evaluate the educational success of the course. The test is written in a language that is accessible to people who have never taken physics courses but the test is quite different from standard exams which students may be able to pass by memorising and applying formulas. The FCI test purely examines conceptual knowledge. It shows that there is no correlation between standard exam results and test results of individual students. Many students do not change their incorrect beliefs about physics concepts when they are taking physics classes. The revelation that traditional teaching methods are largely ineffective (Hake, 1998) led physics professors to search for alternative teaching methods and led to the establishment of Physics Education Research as a field of study. It seems that misconceptions are particularly visible in physics education because, on the one hand, people have naive physics theories about natural laws based on observation and experience and, on the other hand, scientific physics theory describes concepts and laws with mathematical precision which are experimentally verifiable but which sometimes contradict naive observation and experience. Outside the natural sciences, concepts are not usually definable and verifiable with such rigour and precision. But the insights of Physics Education Research should also be relevant for the other natural sciences and mathematics (the latter as discussed by Riegler (2010)).

A conclusion of Physics Education Research is that teaching methods involving interactive engagement (Hake, 1998) tend to be more successful in improving the conceptual understanding of students than traditional teaching methods. Interactive engagement is achieved by questioning and challenging students to think instead of just memorising facts. Several factors appear to be contributing to the success of interactive engagement teaching, including cognitive, social constructive and psychological factors.

From a cognitive viewpoint, it has been known for many years (Auble and Franks, 1978) that effort toward comprehension improves recall, i.e., if someone makes an effort at finding a solution before the solution is presented, recall is higher than if a solution is presented right away. Furthermore, active recall is more beneficial for long-term retention than passive exposure (Ellis, 1995). Conway et al. (1992) report that coursework marks are a better predictor for long-term retention than exam marks, possibly because creating a piece of coursework requires the students to be involved with the subject matter at a deeper level than when they reiterate facts during an exam. Conway et al.'s (1992) paper also confirms other observations of Physics Education Research with respect to other domains: procedural knowledge (where students learn something by doing it) is retained much better than declarative knowledge. Students who take only one course in a subject domain tend to forget it completely after a few years. In particular although they might remember some isolated facts, their understanding of the subject is first to disappear - presumably because they never really understood it in the first place. Students who take several courses on a topic and achieve a certain level of proficiency and understanding will retain a large portion of their knowledge for a long time. Thus if interactive engagement teaching leads to a better understanding of a subject, it will help students to remember what they have learnt more permanently than just until the end of the term.

An example of interactive engagement teaching is Mazur's (1996) peer instruction which is even feasible in large classes. Using peer instruction, a lecturer pauses a lecture with challenging questions which the students discuss among each other. Apparently students do not change their conceptual knowledge just because a teacher provides them with facts or a good explanation or even with a demonstration. But if they discuss questions among each other, the students who do have correct conceptual understanding tend to be able to convince their peers. In addition to the cognitive aspects of interactive engagement learning, there seems to be a social component involved: peer pressure seems to be a stronger motivation for people to question and change their beliefs than explanation or observation.

Last but not least, psychological aspects are involved in learning processes. Devlin (2000) argues that mathematicians are psychologically different to non-mathematicians because mathematicians think about mathematical objects in an emotional, associative manner in the same way as other people think about physical or even animate objects. For example, mathematicians might attribute emotional features to numbers and other abstract objects. Other psychological aspects are involved when people experience clashes between observation and scientific explanations, as for example in optical illusions which are clashes of visual perception and logical, geometrical explanations. Some people perceive clashes as emotionally upsetting. A famous example is the Monty Hall problem[2] about the winning chances in a game show. When Marilyn Vos Savant discussed it and similar problems in her column in the TV magazine Parade, readers responded with angry, emotional letters: "I will never read your column again[3]" or "As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and in the future being more careful[4]." - written by someone with a Ph.D who was wrong! One can speculate that animals have evolved probabilistic intuitions in order to make survival decisions which evoke strong emotional responses when challenged. Another example of intuitions contradicting mathematical probability is the belief which many people have that the longer they have played in the lottery without winning, the more likely it is that they are going to win the next time they play. Again this belief tends to have an emotional component as anybody can observe who has ever discussed it with lottery players.

In summary, teaching a topic which contradicts the existing conceptual structures which the students have is challenging. Methods such as peer instruction can help to overcome cognitive, social constructivist and emotional obstacles. Clearly, not all topics evoke such difficulties and some can be taught with more standard teaching methods. Thus it would be useful for a teacher to know in advance which areas of the subject domain are going to produce conceptual difficulties and which not. McDermott (2001) argues that there is only a limited number of re-occurring conceptual difficulties which tend to be experienced by all students similarly. The idea for this paper is that FCA might provide useful methods for detecting and analysing conceptual difficulties. Although McDermott (2001) emphasises that just detecting misconceptions is not sufficient for improving teaching, providing a better understanding of the conceptual structures of misconceptions is going to be beneficial for teachers. In the following, three examples of conceptual difficulties in mathematics education are analysed using FCA.

3 Conceptual difficulties of the equality sign

Prediger (2010) discusses problems pupils are having with developing an appropriate conceptual model of equality. In primary school, pupils tend to experience the equal sign as a request to calculate something. For example, "$2 + 3 =$" might be printed in a textbook. Prediger calls this the operational use because pupils are requested to perform an operation. Apparently, this can lead to difficulties later when the equal sign is used in its more general algebraic meaning (or its "relational" meaning). For example, Prediger

[2] http://en.wikipedia.org/wiki/Monty_Hall_problem

[3] Parade Magazine, July 27, 1997

[4] http://www.marilynvossavant.com/articles/gameshow.html

quotes the case of a pupil who says that the equation $24 \times 7 = 20 \times 7 + 4 \times 7$ is wrong because "24×7 does not equal 20" and the case of a pupil who writes "$1 \times 10 = 10 + 110 = 120$". Prediger then discusses the difficulties which prospective teachers might encounter in understanding the problems these pupils are having. In her analysis she distinguishes operational, relational and specification uses (such as defining $x := 4$) of the equal sign. She divides the relational use further into symmetric identities ($4 + 5 = 5 + 4$), general equivalences ($(a - b)(a + b) = a^2 - b^2$), searching for unknowns ($x^2 = 6 - x$) and contextual uses ($a^2 + b^2 = c^2$) where the variables are meaningful in a context, such as characterising a right-angled triangle.

To demonstrate the use of FCA in this area we have modelled the problem as a formal context. The formal objects are examples of uses of the equal sign, inequality ($>$) and equivalence ($\Leftrightarrow$). Furthermore we added basic operations from programming languages: not-equal (!=), test for equality (==) and Boolean operators (&&). Four of the formal attributes are from Prediger's classification: "operation", "contextual", "definition" (i.e. specification) and "law" (i.e equivalence). Here, "operation" refers to rule-based drills where the students solve a problem in a precisely taught manner and the equality sign is always read from left to right. A "definition" for other symbols than "=" defines a set of possible values for a variable (e.g., $i > 1$). Furthermore, two attributes have been added which distinguish whether the statements are true for all values of the variables or just for some. Prediger's "unknowns" has been replaced with "test" as a request to evaluate an expression with respect to variables with given values.

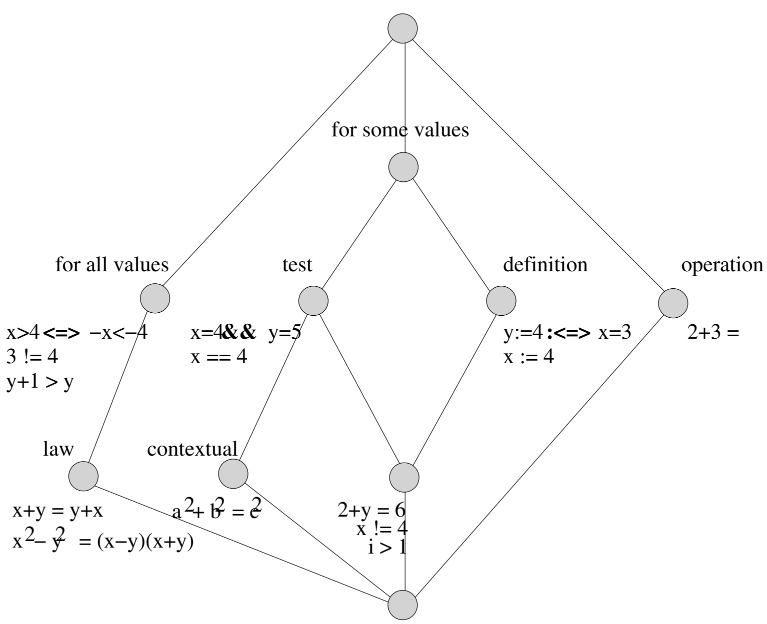

Fig. 1. Equation, assignment and comparison operators

The resulting concept lattice (Fig. 1) shows a classification which is slightly different from Prediger's list[5]. The operational use of the equal sign is not connected to any of the other uses. Although this results directly from the definition of the formal attribute "operation", it represents implicit structure which the authors were not aware of before the lattice was constructed. The separation of "operation" from the other concepts provides a graphical explanation as to why students may find it particularly difficult to progress from an operational use to the more general algebraic use.

The extensions of the concepts under "for all values" contain tautologies. But there is a distinction made between those which students have to specifically learn (under "law") in order to understand how the operators work and those which just happen to be true. There are three different reasons why a statement might only be true for some values of the variables: the variables are defined in the statement; it is to be tested whether the statement is true (or for which variables it is true); and in the contextual use, the statement is only true in some contexts and thus describes such contexts. Some interesting cases are under both "test" and "definition": an equation $2 + y = 6$ is both an implicit definition of y and a request to evaluate which values of y yield the equation to be true. For the use of != and $<$, it depends on the context whether the statements are meant to be evaluated for their truth value or whether they are meant to define a range for their variables.

4 Conceptual difficulties of translating text into algebraic expressions

A well-known conceptual difficulty that mathematics students experience pertains to the translation of text into algebraic expressions. Clement (1982) conducts an experiment where he asks students to write an equation using the variables S and P to represent "there are six times as many students as professors". His findings are that only 40-60% of the students produce a correct answer ($S = 6P$). The most common incorrect answer is $6S = P$. He provides two reasons for the incorrect answer: first, some students simply translate the sentence into mathematical symbols in the same word order. Second, some students use a static comparison pattern or, in other words, an incorrect schema where S and P do not represent numbers, but instead units of students and professors. This is in the same manner as how m and km are used in $1000m = 1km$. In this case, m and km are not variables but represent a fixed "1 to 1000" relationship. It is not possible to substitute arbitrary values for m or km, but m can be substituted with $\frac{1}{1000}km$ and km can be substituted with $1000m$, yielding, for example, $2000m = 2000 \times \frac{1}{1000}km = 2km$. One difference between units and variables for numbers is that it is not usually acceptable to insert a multiplication sign between a number and its unit.

Table 1 summarises the differences between the two conceptual systems: in the first one the letters represent units, in the second, algebraic one the letters represent variables for numbers. The first conceptual system has a meronymic (part-whole) quality. A certain, fixed aggregate of the smaller units constitutes the larger unit. The extension

[5] In Fig. 1, in the statements with more than one operator the relevant one is printed in bold face.

of $6s = 1p$ is really a fixed “6 to 1” relation which is expressed in s and p. In contrast, the algebraic conceptual system represents normal algebraic use of variables. The extension of $s = 6 * p$ consists of the pairs of values that can be substituted for s and p. The table also shows examples of intensionally equivalent and implied expressions. In the meronymic conceptual system, s is indeed smaller than p because it represents the unit “student” which somehow has less value than the unit “professor”. It is possible to interpret the units s and p as algebraic variables but not as “numbers of”. For example, s could represent the money paid by a student and p the money earned by a professor.

Table 1. Two conceptual systems for the use of letters in equations

the letter means:	unit	variable for number
conceptual system:	meronymic	algebraic operation
representation:	$6s = p$	$s = 6 * p$
extension:	relation: 6 to 1	substitution: $\{(6,1),(12,2),(18,3),...\}$
intensionally equivalent:	$s = \frac{1}{6}p$	$s/p = 6/1$
intensionally implied:	$s < p$	$s > p$

It should be emphasised that both conceptual systems in Table 1 are consistent. In everyday experience, meronymic, unit-based conceptual systems may be much more common than algebraic ones. Thus it should be expected that students who have not yet made much progress towards learning algebra or people who have not recently used algebra would prefer the meronymic, unit-based representation. Ben-Ari (1998) argues that from a constructivist educational viewpoint, students always already have existing mental models which may contradict scientific models. Teachers need to understand the students’ mental models and to build on them instead of discarding them as simply being incorrect. In this case the algebraic use of variables must be taught to people who already employ a different, meronymic conceptual system. They need to learn to use the different systems in different circumstances.

As a further analysis, we have coded the data from two student interviews (Clement, 1982) in a content analytic manner and converted them into a formal context. The formal objects are mathematical notations as used or implied by the students. The formal attributes are verbal descriptions made by the students converted into a slightly more formal language. A cross in the formal context means that the student used a verbal expression with respect to a mathematical notation. Mathematical notations and verbal expressions that were used algebraically incorrectly by a student have been highlighted in bold face.

The following observations can be made from the resulting concept lattice in Fig. 2. Even though it was argued that the two conceptual systems in Table 1 are both consistent conceptual systems, in Fig. 2 it appears that the correct statement $s = 6p$ is conceptually better refined than the incorrect one $6s = p$ which is more isolated in the lattice. This is because the lattice combines the data from two student interviews: one student with a correct answer of the problem who provided detailed explanations and reasons for why his answer was correct and another student who produced an incorrect

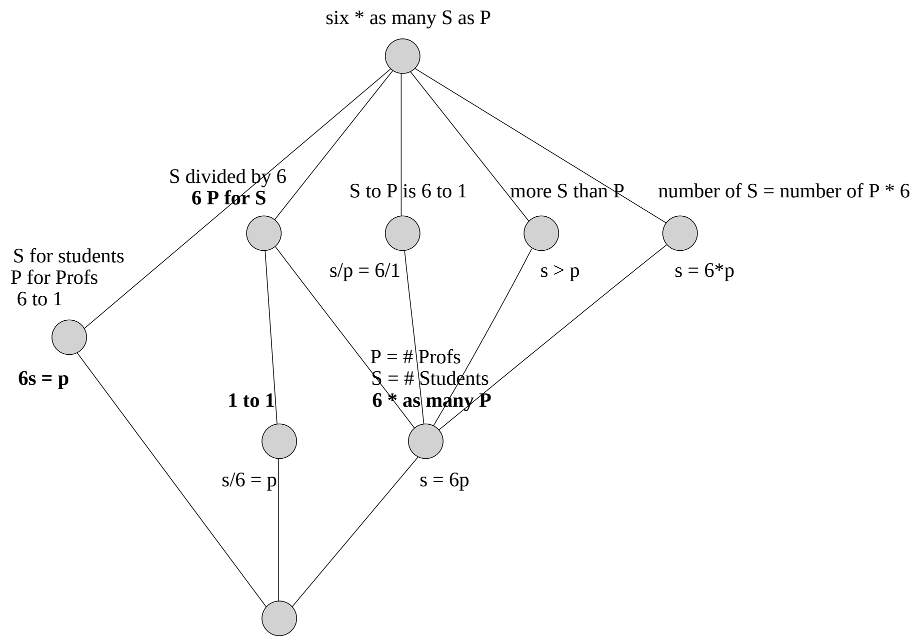

Fig. 2. Data from student interviews

solution which does not appear to be very coherent according to the lattice. The student with the correct answer understood that the variables represent "numbers of". The other student said that "S stands for student". It is interesting to observe that the student with the incorrect solution focussed more on the relationship ("6 to 1" and "1 to 1") which is indeed the extension of the incorrect representation according to Table 1. When he talked about "1 to 1" he really meant to express a "fixed correspondence". The student with the correct answer on the other hand demonstrated detailed understanding of algebraic transformations which is why his arguments contained intensionally equivalent and implied statements.

Clement (1982) observes that different, unsuccessful strategies have been tried to help students in finding correct solutions. It is our opinion that all of the strategies mentioned by Clement are methods from within the algebraic conceptual system (for example telling the student to substitute numbers for variables or to determine whether there are more students or professors). Presumably all instructors involved in the experiments were of the opinion that a student's attempt was plain wrong, not that it was part of an internally coherent, but different conceptual system. One can speculate what would happen if the students were somehow taught that there are different conceptual systems for use of letters in equations and how to determine which conceptual system is appropriate for which problem. We suspect that in general in most basic mathematics teaching the modelling aspect (how to determine which type of solution belongs to which type of problem) is not significantly highlighted. Thus most students will not be aware that there are different conceptual structures involved in using mathematics and will not have been taught to analyse their strategies from that aspect on a meta-level. They might be aware that they are not "very good at mathematics" without knowing any reasons for the difficulties encountered.

5 Conceptual difficulties of the notion of "function"

The third example discussed in this paper refers to the conceptual difficulties encountered by students in learning the notion of "function". The problem is well-known and has been discussed numerous times (e.g., Leinhardt et al. (1990) and Breidenbach et al. (1992)). Quite often students can recall a correct formal definition of a function, but misconceptions become obvious when they are asked to determine whether something can be represented as a function or not. Leinhardt et al. provide the following list of misconceptions:

- Too narrow understanding of "function". Only functions with certain characteristics (regularity, symmetry, linearity, one-to-one, causal relationship, etc) or which are represented in a certain manner (formula, graph, table) are recognised as functions.
- Correspondence: students often believe that functions must be one-to-one and they might be confused about the difference between many-to-one and one-to-many.
- Linearity: students have a tendency towards linearity. They tend to prefer straight lines in graphs.
- Continuous versus discrete: historically, functions were not allowed to be discontinuous. Students have problems understanding the notion of continuity. They discretise continuous data.
- Representations: problems translating graphs into formulas and vice versa.
- Interpretation of graphs: students have problems with confusing intervals and points, slope and height. They might interpret graphs in a literal, iconic manner.
- Variables: students have problems with the notion of "variable". Some do not accept constant functions as functions.
- Notation: students have problems understanding axes and scales in a graph.

Breidenbach et al. (1992) emphasise the "process conception of function". They argue that an understanding of "function" proceeds from a pre-function over an action to a process stage. In our opinion the notion of "process" is misleading in this case because it implies a temporal progression which is not involved in functions. This is in contrast to functions implemented in a computer where an input is converted into an output in real time so that the output is generated temporally after the input has been processed and the input may be purged from memory after it has been used. Although both Clement (1982) and Breidenbach et al. observe that students often develop a better understanding of mathematical operations if they execute them as computer programs, many features of mathematical objects cannot be adequately represented on a computer (for example infinity) and thus there are limits to the use of computer programs for representing mathematical ideas.

In our opinion, it is not the "process conception of function" that is relevant but instead simply the "concept of function". Breidenbach et al.'s tests for whether students understand the notion of function include: asking students to provide a definition (i.e., an intensional description); asking students to decide whether something is a function or not (i.e., evaluating whether something is in the extension of "function"); and asking students to perform operations with functions (composition and reversion) which demonstrates an understanding of the implied intensional features. Thus all of the tests are aimed at demonstrating whether or not students have an acceptable concept of function, including extension, intension, subsumption, implication and equivalence. Initially, students appear to have incomplete or disconnected concepts of "function". For example, Breidenbach et al. report that the examples of functions provided by students are more sophisticated than their definitions whereas Leinhardt et al. (1990) state that students can recall an accurate definition of "function" but cannot apply it. In either case there is a mismatch between the extension and intension of "function".

Breidenbach et al. also test whether students develop an abstract understanding of particular functions. For example, they define a complicated function F(a)(b)(c) the meaning of which is "the cth character in the string which is the name of the integer given by the ath power of the integer b". We would argue that although translating a function from one representation (formula) to another (textual representation) is an important aspect of using functions, this particular example is really more a test for intelligence than for an understanding of the notion of "function". Similarly, they use strings as examples of functions. Depending on how much experience students have with programming languages, they may or may not be familiar with the representation of strings as arrays of characters. Thus, interpreting a string as a function depends on the students' programming knowledge, not on an understanding of function. On the other hand, if students do have a programming background, then functions from programming languages can be used to emphasise to students that not all functions are of the form "$f(x) =$".

From an FCA viewpoint, data collected from student interviews and exams can be represented as concept lattices to visualise such conceptual differences. The following attributes are examples of how to characterise an understanding of "function".

- representation: set, equation, graph, verbal description, table, computer program, ...
- constant, linear, quadratic, ...
- causal, non-causal
- discrete, continuous
- 1-to-1, 1-to-many (i.e., the reverse is a function), many-to-1, many-to-many
- finite domain, infinite domain

In Fig. 3 some of these attributes are selected as formal attributes and applied to examples of functions as formal objects. The lattice shows a conceptual structure of "function". In this case, continuous functions with infinite domains tend to have more attributes. For computer programs, it is a matter of choice whether one considers "min(x)" as an abstract function with an infinite domain of possible values x (where x is a set or other container object) or as an actually implemented function with a finite domain. The use of such an expert-designed concept lattice is in comparison with lattices obtained from student data (which we have not included in this paper).

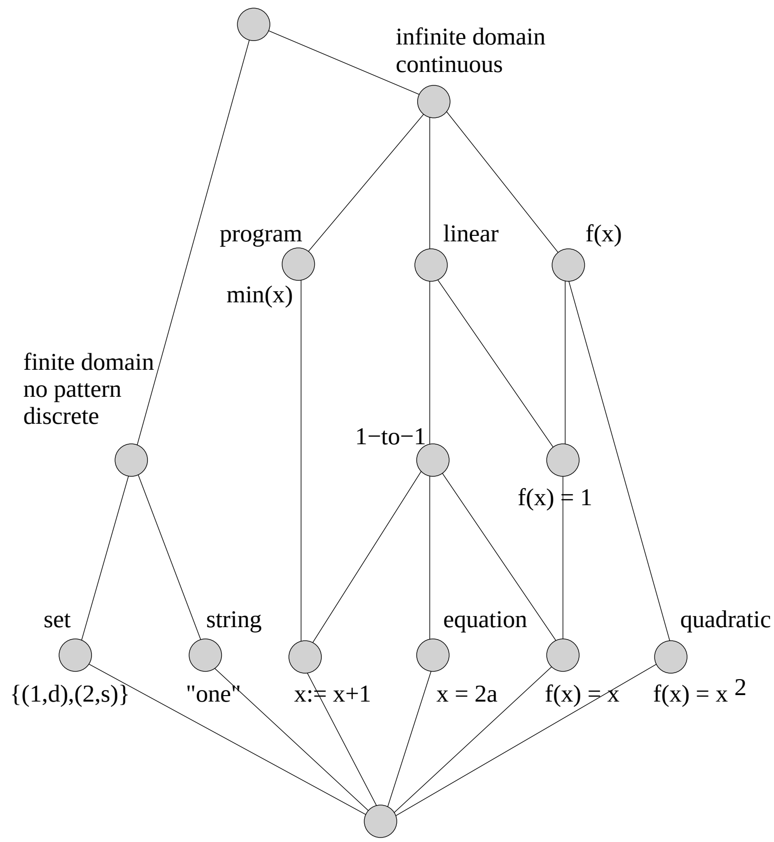

Fig. 3. Attributes of functions

6 Conclusion

This paper argues that FCA provides useful methods for analysing conceptual difficulties in learning processes. Teachers can use the construction of formal contexts and concept lattices in order to explore the implicit structures in mathematical notions. The data for the lattices can be obtained from the literature on misconceptions, from student interviews or from assessment data. In this manner FCA becomes a tool for exploration and for making underlying assumptions explicit.

Traditional teaching methods are often not successful in teaching conceptually challenging topics. While teaching methods have been developed that are more promising, teachers still need to know when to apply such methods. Thus they need to determine what the specific conceptual challenges are with respect to a certain domain. FCA can be employed as a tool by teachers to familiarise themselves with the materials and to structure difficult topics. Based on the improved understanding of the topics, teachers can then design interactive engagement teaching exercises that focus on the conceptually challenging aspects which the students need to learn.

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